Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 41 (2012) 643 – 649

Engineering Procedia

www.elsevier.com/locate/procedia

International Symposium on Robotics and Intelligent Sensors 2012 (IRIS 2012)

Robust Control of Bipedal Humanoid (TPinokio)

Wee Teck Chew [a,*] , A.Astolfi[a] , Xie Ming[b]

[a]Imperial College London, U.K.
[b]Nanyang Technological University, Singapore

Abstract

A stable walking motion requires effective gait balancing and robust posture correction algorithms. However, to develop and implement such intelligent motion algorithms remain a challenging task for researchers. In order to minimize the modeling errors and disturbances, this paper presents an alternative approach in generating a stable Centre-of-Mass (CoM) trajectory by applying augmented model predictive control. The propose approach is to apply Augmented Model Predictive Control (AMPC) algorithm with on-line time shift and look ahead to process future data to optimize a control signal by minimizing a cost function so that the system is able to track the reference Zero Moment Point (ZMP) as close as possible, and at the same time to limit the motion jerk in order to improve the robot walking stability.

Keywords: humanoid robot, augmented model predictive control, stability.

1. Introduction

The developments of bipedal humanoid robots have begun more than thirty years ago. Despite the numerous amounts of research efforts, to control a bipedal robot is still an extremely challenging task in terms of adaptability, robustness and stability. The most common approach is the Linear Inverted Pendulum Model (LIPM) and Zero Moment Point(ZMP) method [3][4]. The problem with the LIPM method is that it is a non-minimum phase system [6], which produces a undershoot response. In order to compensate for this undesired effect, a new approach with Augmented Model Predictive Control (AMPC) method is proposed in this paper. This method reduces the landing impact force and improves the performance of ZMP tracking under external disturbances. The effectiveness of the proposed scheme is demonstrated in simulation and in comparison with the preview control method.

2. Robot Mechanical Structure

TPinokio is approximately 1.5 meter tall and weighs approximately 45kg. It has more than 25 D.O.F. As depicted in Fig. 1. and Fig, 2. It is designed with light weight joint-link with negligible inertia, and with compact-joint housing mechanism. It has a simple kinematics structure and it can be modeled as a simplified 'point-mass' system [5]. The robot's feet are fitted with force sensors to measure the actual ZMP location. The measured ZMP value is used to determine the system tracking error and to provide a feedback signal to an observer [2] to improve the system stability.

* Corresponding author.
E-mail address: t.wee10@imperial.ac.uk

1877-7058 © 2012 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
doi:10.1016/j.proeng.2012.07.224

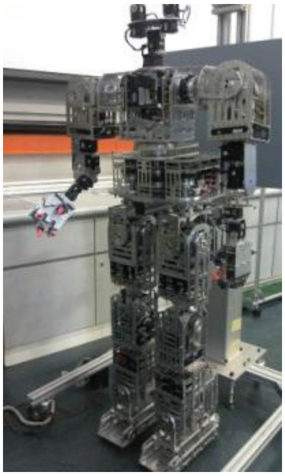


Fig. 1. TPinokio Robot

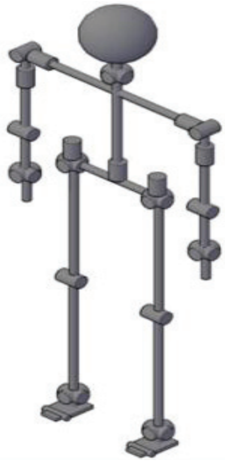


Fig. 2. TPinokio stick structure

3. Robot Model

3.1. Inverted Pendulum Model

In order to define the relationship of the ZMP and the CoM trajectories, a simplified Linear Inverted Pendulum Model (LIPM) is studied in this paper. During walking, in the leg swing phase, the CoM of the robot may be modeled as a 'point mass'[5] and it is assumed to be connected to the stance foot like an inverted pendulum on a moving base, as depicted in Fig. 3. In order to define the relationship of the ZMP and the CoM trajectories, a simplified ZMP (x_zmp,y_zmp,0) as shown in Fig. 4., with Inverted Pendulum Model (IPM) may be computed.

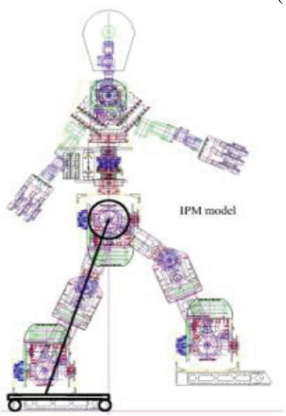


Fig. 3. TPinokio LIPM model

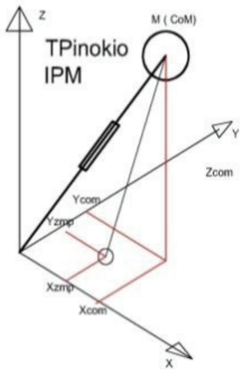


Fig. 4. ZMP coordinate

The Linear Inverted Pendulum Model (LIPM) equations under constraint control (move along a defined plane) are given as [3]:

$$x_{zmp} = x_{com} - \frac{z_{com}}{g}\ddot{x}_{com} \quad , \quad y_{zmp} = y_{com} - \frac{z_{com}}{g}\ddot{y}_{com} \tag{1}$$

The above equations are decoupled, and they can be expressed independently. With an extra variable (system jerk) introduced as:

$$u_x = \frac{d}{dt}\ddot{x}_{com} = \dddot{x}_{com} \quad , \quad u_y = \frac{d}{dt}\ddot{y}_{com} = \dddot{y}_{com} \tag{2}$$

The trajectories of the CoM and of the ZMP are discretized, with intervals of constant lengths T, and $t = k$T, with k =1, 2 ,…, yielding :

$$\hat{x}_{k}=\begin{bmatrix} x_{com}(kT) \\ \dot{x}_{com}(kT) \\ \ddot{x}_{com}(kT) \end{bmatrix}, \dddot{x}_k = u_x(kT), z_k^x = x_{zmp}(kT) \qquad \hat{y}_{k}=\begin{bmatrix} y_{com}(kT) \\ \dot{y}_{com}(kT) \\ \ddot{y}_{com}(kT) \end{bmatrix}, \dddot{y}_k = u_y(kT), z_k^y = y_{zmp}(kT)$$

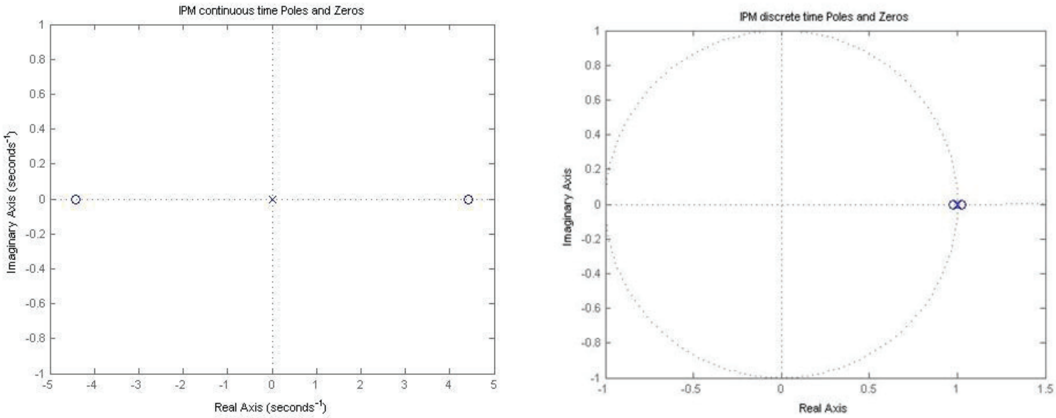
$$\hat{x}_{k+1} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \hat{x}_k + \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} \dddot{x}_k \qquad \hat{y}_{k+1} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \hat{y}_k + \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix} \dddot{y}_k$$

$$z_k^x = \begin{bmatrix} 1 & 0 & -z_{com}/g \end{bmatrix} \hat{x}_k \qquad z_k^y = \begin{bmatrix} 1 & 0 & -z_{com}/g \end{bmatrix} \hat{y}_k \tag{3}$$

The set of equations (3) can be used to determine the dynamic behaviour of the LIPM system.

3.2. Characteristics of a Linear Inverted Pendulum Model

The LIPM is a non-minimum phase system, there is a undesired zero at the RHP, as shown in Fig. 5 (a) & (b).The undesired zero appeared outside the unit circle. The RHP zero will produce a reverse action undershoot transient response [6] to a step input. Care should be taken to address the initial 'water-bed' effect, as this may cause the robot to be unstable.



(a) (b)

Fig. 5. LIPM S-plane and Poles-Zeros Plot

4. Augmented MPC State-Space Model with Embedded Integrator

Model predictive control (MPC) is designed based on a mathematical model of the plant system. In this paper, state-space model is used in the tracking system design. The current time state variables are used to perform the predicting ahead operation [7]. The state-space model of the bipedal robot is presented in this section.

4.1. The Augmented Model Predicted Control (AMPC)Model

The proposed AMPC model used in this paper is an augmented state-space model with embedded integrator. The set of equations (3) is re-written into the standard state-space form:

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) \\ y(k) &= C_m x_m(k) \end{aligned} \tag{4}$$

where x_m is the state variable (CoM) , u is the control input variable (jerk), y is the tracked ZMP output. In order to include an integrator, by taking a difference operation on both sides of (4), yields:

$$x_m(k+1) - x_m(k) = A_m(x_m(k) - x_m(k-1)) + B_m(u(k) - u(k-1)) \tag{5}$$

denoting the difference in the state variable and the difference in the control variable by $\Delta x_m(k+1) = x_m(k+1) - x_m(k)$; $\Delta x_m(k) = x_m(k) - x_m(k-1)$; $\Delta u(k+1) = u(k) - u(k-1)$; and $\Delta x_m(k+1) = A_m \Delta x_m(k) + B_m \Delta u(k)$

. And define a new state variable vector to be $x(k) = [\Delta x_m(k)^T \quad y(k)]^T$,This yields the augmented state-space model [7]:

$$\overbrace{\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix}}^{x(k+1)} = \overbrace{\begin{bmatrix} A_m & O_m^T \\ C_m A_m & 1 \end{bmatrix}}^{A} \overbrace{\begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}}^{x(k)} + \overbrace{\begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}}^{B} \Delta u(k)$$

$$y(k) = \overbrace{\begin{bmatrix} O_m & 1 \end{bmatrix}}^{C} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix} \tag{6}$$

Where $O_m = [0\ 0\ 0]$, is the dimension of the state variable. The triplet (A, B, C) is the augmented model.

4.2. Prediction of CoM (the state) and ZMP (the output variable)

The system jerk (the control trajectory) is denoted as:

$$\Delta u(k_i)\ ,\ \Delta u(k_i+1)\ , \dots ,\ \Delta u(k_i+N_c-1) \tag{7}$$

Where N_c is the control horizon, used to capture the future control system jerk trajectory. where N_p is the length of the prediction horizon optimization window. The future CoM state variable are predicted for N_p samples. Denote the future CoM state variables as:

$$x(k_i+1|k_i), x(k_i+2|k_i), \dots, x(k_i+m|k_i), \dots, x(k_i+N_p|k_i) \tag{8}$$

where $x(k_i+m|k_i)$ is the predicted state variable at (k_i+m) with given current plant information $x(k_i)$. The control horizon N_c is chosen to be $N_c \le N_p$ to the prediction horizon N_p. All the predicted variables are in terms of current state variable information $x(k_i)$ and the future control parameters $\Delta u(k_i+j)$, where, j = 0, 1, . . .N_c − 1. Define the following vectors:

$$\begin{aligned} Y &= [y(k_i+1|k_i) \quad y(k_i+2|k_i) \quad y(k_i+3|k_i) \dots \quad y(k_i+N_p|k_i)]^T \\ \Delta U &= [\Delta u(k_i) \quad \Delta u(k_i+1) \quad \Delta u(k_i+2) \dots \quad \Delta u(k_i+N_c-1)]^T \end{aligned} \tag{9}$$

The dimension of Y and U is N_p and N_c respectively. By rearranging the parameters into matrix form :

$$\mathrm{Y} = Fx(k_i) + \Phi \Delta U$$

$$F = \begin{bmatrix} CA \\ CA^2 \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}; \quad \Phi = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & \vdots & 0 \\ CA^2B & CAB & CB & \cdots & 0 \\ \vdots & & & & \\ CA^{N_p-1}B & CA^{N_p-2}B & CA^{N_p-3}B & \cdots & CA^{N_p-N_c}B \end{bmatrix} \tag{10}$$

4.3. ZMP Tracking and Optimization

This optimization process is then translated to find an optimal control vector U such that the error between the input reference ZMP set-point trajectory and the CoM output trajectory is minimized. The vector that contains the set-point data information is written as:

$$R_s = [\overbrace{1 \quad 1\ldots \quad 1}^{N_p}]\, r(k_i + t_a) = \bar{R}_s r(k_i + t_a) \tag{11}$$

where t_a is the adjustable look forward time-shift constant. The cost function J is defined as:

$$J = (R_s - Y)^T (R_s - Y) + \Delta U^T \bar{R} \Delta U \tag{12}$$

Where the first term is linked to the objective of minimizing the errors between the predicted ZMP output and the reference ZMP set-point, and the second term reflects the consideration given to the rate of change in the system jerk ΔU when the objective function J is made to be as small as possible. The diagonal matrix $\bar{R} = r_w I_{N_c \times N_c} (r_w \geq 0)$ is used as a tuning parameter to improve the closed-loop performance. For the case that $r_w = 0$, the goal would be solely to make the ZMP error $(R_s - Y)^T (R_s - Y)$. as small as possible. For the case of large r_w, the cost function is interpreted as it is carefully consider how large the rate of change in the system jerk (ΔU) might be and cautiously reduce the ZMP error. To determine the optimal ΔU that minimize J, substitute equation (10) into (12), J is expressed as:

$$J = (R_s - Fx(k_i))^T (R_s - Fx(k_i)) - 2\Delta U^T \Phi^T \big(R_s - Fx(k_i)\big) + \Delta U^T (\Phi^T \Phi + \bar{R}) \Delta U \tag{13}$$

The necessary condition of the minimum J is derived:

$$\frac{\partial J}{\partial \Delta U} = -2\Phi^T \big(R_s - Fx(k_i)\big) + 2(\Phi^T \Phi + \bar{R})\Delta U = 0 \tag{14}$$

Re-arrange equation (14) and substitute (11) into (14), from which the optimal solution for the control signal is obtained:

$$\Delta U = (\Phi^T \Phi + \bar{R})^{-1} \Phi^T \big(\bar{R}_s r(k_i + t_a) - Fx(k_i)\big) \tag{15}$$

As such, the optimal solution of the control signal is linked to the ZMP reference set-point signal and the state variable (CoM) by the above equation (15).

4.4. Simulation

The initial simulation shows good tracking of the desired ZMP & CoM trajectory, as shown in Fig 6. The simulation parameters are given as follows: $N_c = 100, N_p = 300, t_a = 80, r_w = 10^{-5}$ .The system is able to shift its desired CoM (red cure) to left/right feet during straight walking. Although AMPC do exhibits initial undershoot (water-bed effect) due to the inherent characteristics of IPM because of the RHP zero, but the CoM output (the desired signal in red) is smooth and it is able to follows closely to the reference ZMP (blue waveform) trajectory. Fig. 7 shown that the jerk in both X and Y direction is minimized to zero by the cost function.

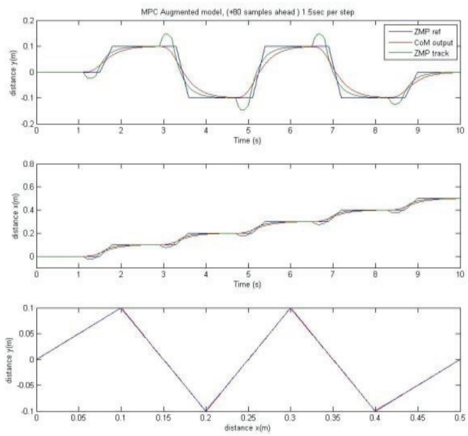


Fig. 6. AMPC CoM trajectory

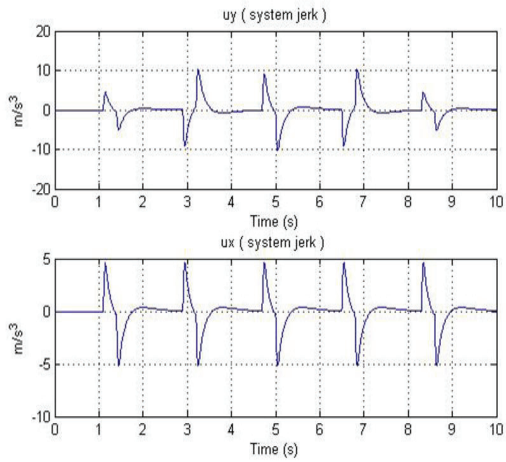


Fig. 7. System Jerk reduced to zero.

4.5. Comparison of AMPC and Preview control method

With closer examination of the CoM trajectory produced using the AMPC tracking as depicted in Fig. 8. It can be seen that the CoM curve (in red) moves slowly toward the reference ZMP curve (in blue). This can be interpreted as the controller is trying to reduce the jerk and impact force during swing foot landing and cautiously reduce the ZMP error. As a result, the weight of CoM transfers from the supporting foot to the landing foot (new supporting foot) during the double support phase (DSP) is slow and gentle. As a comparison with the preview control technique[6], which produces a symmetrical sinusoidal waveform (in green), the AMPC algorithm produce a truncated sinusoidal trajectory during swing foot landing due to its 'soft landing fast take off' characteristic. The other advantages of using AMPC model is that the optimization process does not need to solve Riccati equation, and the optimization window can be easily adjusted to fine-tune the system response.

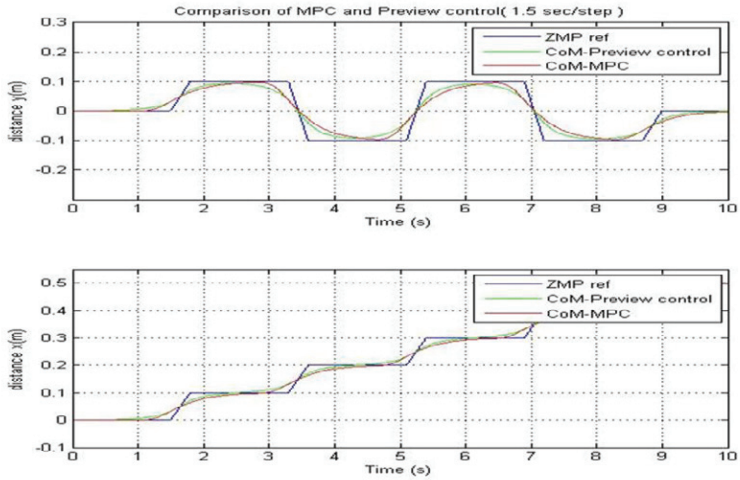


Fig. 8. Comparison between the AMPC and the Preview Control waveform

4.6. Augmented MPC model with noise

One of the key features of AMPC predictive control is the ability to handle constraints in the design. To check the robustness of the AMPC model in tracking the desired ZMP, a random noise (±0.05m) is added to the reference ZMP waveform. The simulation results are as depicted in Fig. 9. It shows that the AMPC model is still able to produce good track output under a noisy environment.

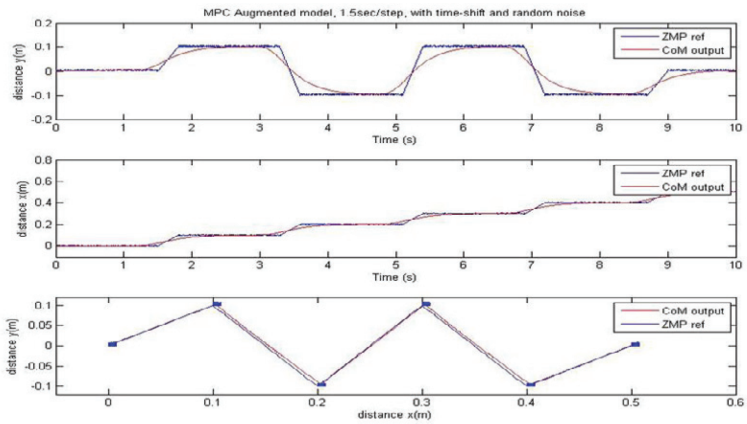
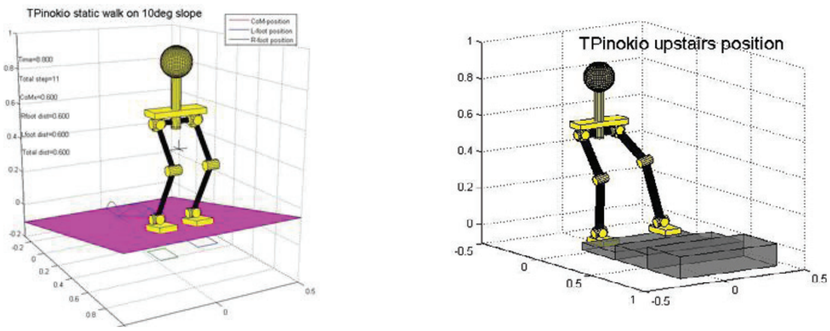


Fig. 9. CoM trajectory with external disturbance

4.7. Augmented MPC model on slope and stairs

The AMPC algorithm can also be applied to control a robot walking on slope, on stairs and clearing of obstacle, as depicted in Fig. 10. (a) & (b). The joints' trajectory of straight walking for 2 steps on flat surface is as shown in Fig. 11.



(a) Walking up a 10^{o} slope (b) Walking on stairs

Fig. 10. TPinokio walking on a slope and stairs

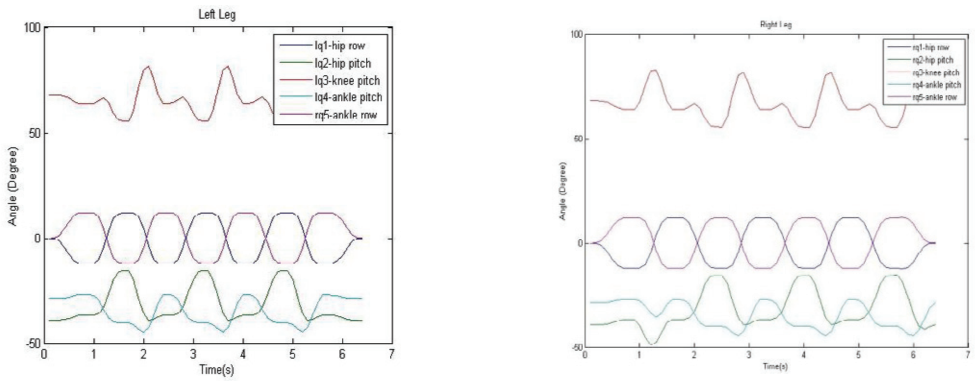


Fig. 11. TPinokio joints' trajectory (for 2 steps) on flat surface

5. Conclusion

In this paper, a new control algorithm with AMPC method is proposed and the simulation results are presented and compared with the preview control method. The future work is to combine the discrete-time AMPC with Laguerre function and observer [2] to further enhance and refine the tracking algorithm and joints' trajectory in order to reduce the effect of the undershoot water-bed phenomenon.

References

[1] M. Xie, G.Q. Zhang, H. Ying, "Planning and Control of Biped Walking along Curved Paths on Unknown and Uneven Terrain", ICIRA 2009, Springer-Verlag, 2009, pp. 1032-1043.

[2] A. Astolfi, R. Ortega, A. Venkatraman, Global observer design for mechanical systems with non-holonomic constraints, American Control Conference (ACC), 2010, pp. 202-207.

[3] S. Kajita, F. Kahehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi, H. Hirukawa, Biped Walking Pattern Generation using Preview Control of the Zero-Moment-Point, Vol. 2, IEEE International Conf on Robotics and Automation, September 2003, pp. 1620-1626.

[4] M.Vukobratovic, B. Borovac, ZMP - Thirty five years of its life. International Journal of Humanoid Robotics, 2004, pp. 157–173.

[5] Teck-Chew Wee, A. Astolfi, Ming Xie, 'Modular-Joint Design of a TPINOKIO Bipedal Robot', ICINCO 2011.

[6] Jesse B.H., Dennis S.B., Nonminimum-Phase Zeros : Much To Do About Nothing, Part II, Humanoid Robots, IEEE Control System Magazine, Jine 2007, pp. 45-57.

[7] L. Wang, Model Predictive Control System Design and Implementation using MATLAB, Book, Springer 2009.